CPT-like states in an ensemble of interacting fermions. On the possibility of new mechanism of superconductivity

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Using the standard Hamiltonian of the BCS theory, we show that in an ensemble of interacting fermions there exists a coherent state $|NC\rangle$, which nullifies the Hamiltonian of the interparticle interaction. This state has an analogy with the well-known in quantum optics coherent population trapping effect (CPT). A possible application of such CPT-like states in the superconductivity theory is discussed.

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INTRODUCTION

The effect of coherent population trapping (CPT) (see [1, 2, 3, 4, 5] and references therein) is one of nonlinear interference effects. Owing to number of its manifestations in different optical phenomena and its practical applications CPT occupies one of leading place in modern laser physics, nonlinear and quantum optics. For example, CPT is used in high-resolution spectroscopy [6], nonlinear optics of resonance media [7, 8], laser cooling [9], atom optics and interferometry [10], physics of quantum information [11, 12, 13].

In the case of classical resonant field the CPT theory has been developed for a three-state model [3, 5] as well as for multi-level systems with account for the level degeneracy [14, 15, 16]. Recently we generalize this theory to the case of an ensemble of atoms interacting with a quantized light field [17, 18].

From the very general point of view the essence of CPT can be formulated as follows. Consider two quantum systems (particles or fields) A and B. The interaction between them is described by the Hamiltonian $\widehat{V}_{A-B}$. Then the CPT effect occurs when there exists a nontrivial state $|NC\rangle$, which nullifies the interaction:

$$\widehat{V}_{A-B}|NC\rangle = 0\,. \tag{1}$$

In this state, obviously, the energy exchange between the systems A and B is absent. However, information correlations of the systems can be very strong, leading to important physical consequences. Note that if the system A is equivalent to the system B, then the condition (1) means the absence of the field self-interaction or of the interparticle interaction

$$\widehat{V}_{A-A}|NC\rangle = 0\,. \tag{2}$$

From this general viewpoint the standard CPT effect in the resonant interaction of atoms with electromagnetic field is deciphered as follows: A and B is an ensemble of atoms and resonant photons, respectively; $\widehat{V}_{A-B} = -(\hat{\mathbf{d}}\mathbf{E})$ is the dipole interaction operator, and $|NC\rangle$ is the dark state $|dark\rangle$:

$$-(\hat{\mathbf{d}}\mathbf{E})|dark\rangle = 0\,. \tag{3}$$

In the course of the interaction atoms are accumulated in the dark state, after that they do not scatter light, and they are not scattered by light. The information on various parameters of the resonant field has been encoded in the state $|dark\rangle$ [15, 17].

Our standpoint consists in the following. The CPT principle, expressed by (1) or (2), is universal enough and it can be manifested in various branches of physics. For the first time such a generalized approach to CPT has been developed in our early paper [19], where it is pointed out that from a phenomenological viewpoint the CPT effect is similar to the superconductivity. In [19] the following comparison is carried out: atoms and electromagnetic field from one side, electrons and phonons form the other side. Indeed, a gas of atoms being in the dark state $|dark\rangle$ do not interact with photons (see eq.(3)), similarly to electrons in a superconducting state in solids, which are not scattered by the phonon oscillations of a lattice. In the paper [19] a hypothesis on the possibility of an alternative (to the standard BCS theory [20]) mechanism of superconductivity has been proposed. Namely, a quantum system of electrons and phonons coupled by the interaction Hamiltonian $\widehat{V}_{e-phonon}$ was considered. According to [19], the new mechanism of superconductivity could be based on the existence of such a state $|NC\rangle$, which nullifies the interaction operator $\widehat{V}_{e-phonon}$:

$$\widehat{V}_{e-phonon}|NC\rangle = 0\,, \tag{4}$$

analogously to eq.(3). However, the explicit form of the state $|NC\rangle$ was not found in [19].

In the present paper for the standard Hamiltonian of interparticle interaction in the BCS model [20] we find in explicit and analytical form a CPT-like state of the type (2). A possible application of the obtained results in the superconductivity (or superfluidity for ^{3}He) theory is discussed.

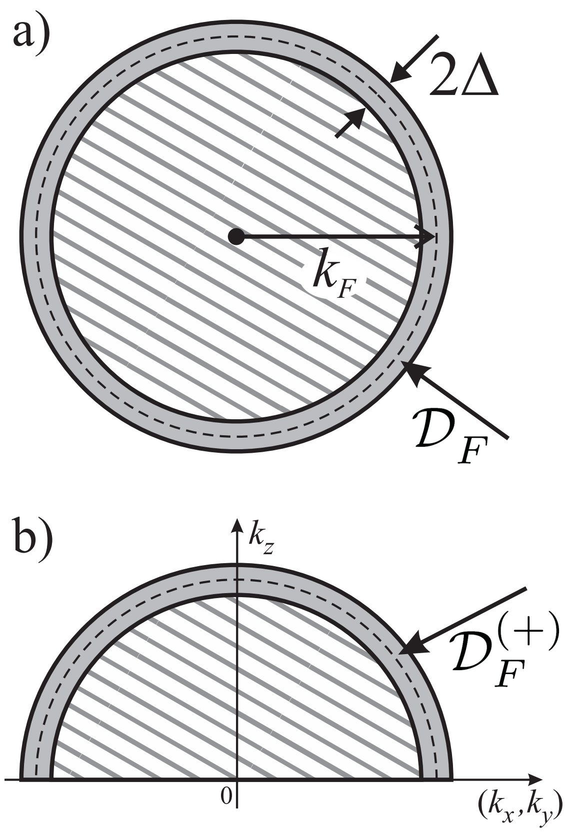

FIG. 1: Illustrations: a) thin spherical layer $\mathcal{D}_F$ with the width 2Δ around the Fermi surface with the radius k_F; b) upper hemispherical layer $\mathcal{D}_F^{(+)}$ $k_z>0$.

ENSEMBLE OF FERMIONS IN A FINITE VOLUME

Consider an ensemble of fermions in a volume L^3. We will use the standard BCS Hamiltonian [20]:

$$\widehat{H} = \widehat{H}_0 + \widehat{W}\,. \tag{5}$$

The Hamiltonian of free particles can be written as:

$$\widehat{H}_0 = \sum_{s,\mathbf{k}} \varepsilon_{\mathbf{k}}\, \hat{a}^\dagger_{s\mathbf{k}} \hat{a}_{s\mathbf{k}}\,, \tag{6}$$

where $\hat{a}^\dagger_{s\mathbf{k}}(\hat{a}_{s\mathbf{k}})$ is the creation (annihilation) operator of a particle in the state with wavevector $\mathbf{k}$ and spin $s = \uparrow, \downarrow$, and $\varepsilon_k = (\hbar\mathbf{k})^2/2m$ is the energy of this state. If the chemical potential μ is introduced into the theory, then $(\varepsilon_{\mathbf{k}} - \mu)$ should be used in (6) instead of $\varepsilon_{\mathbf{k}}$.

The interaction between particles is described by the Hamiltonian coupling particles with opposite momenta and spin:

$$\widehat{W} = \frac{g}{L^3} \sum_{\mathbf{k}_{1,2} \in \mathcal{D}_F} G(\mathbf{k}_1, \mathbf{k}_2)\, \hat{a}^\dagger_{\uparrow\mathbf{k}_1} \hat{a}^\dagger_{\downarrow-\mathbf{k}_1} \hat{a}_{\downarrow-\mathbf{k}_2} \hat{a}_{\uparrow\mathbf{k}_2}\,, \tag{7}$$

$$\mathcal{D}_F: \quad k_F - \Delta \le |\mathbf{k}| \le k_F + \Delta\,.$$

Only particles with wavevectors in the thin layer of the width 2Δ around the Fermi surface (see in Fig.1a), having the radius k_F ($\Delta \ll k_F$), are involved in the interaction. This subset in the wavevector space will be referred to as $\mathcal{D}_F$. If even one of the vectors $\mathbf{k}_{1,2}$ does not belong to the subset $\mathcal{D}_F$, then $G(\mathbf{k}_1,\mathbf{k}_2)$=0. The sign of the interaction constant g in (7) governs the attraction ($g<0$) or repulsion ($g>0$) between particles. The formfactor $G(\mathbf{k}_1,\mathbf{k}_2)$ obeys to the general symmetry condition

$$G(\mathbf{k}_1, \mathbf{k}_2) = G(-\mathbf{k}_1, \mathbf{k}_2) = G(\mathbf{k}_1, -\mathbf{k}_2)\,. \tag{8}$$

It is usually assumed that $G(\mathbf{k}_1,\mathbf{k}_2)$=1 at $\mathbf{k}_{1,2} \in \mathcal{D}_F$. Recall that, according to the standard conception, the model Hamiltonian (7) is determined by the interaction of electrons with the phonons of lattice and Coulomb repulsion between electrons.

It turns out that the operator (7) allows the existence of the CPT-like state $|NC\rangle$, obeying the condition

$$\widehat{W}|NC\rangle = 0\,. \tag{9}$$

Let us build up this state. Consider first the following operator construction:

$$\hat{b}^\dagger_{\mathbf{k}}(\lambda) = \hat{a}^\dagger_{\uparrow\mathbf{k}} \hat{a}^\dagger_{\downarrow-\mathbf{k}} + \lambda\, \hat{a}^\dagger_{\uparrow-\mathbf{k}} \hat{a}^\dagger_{\downarrow\mathbf{k}}\,, \tag{10}$$

which generates two-particle coupled states with opposite wavevectors $\mathbf{k}$ and $-\mathbf{k}$, and with zero total spin; the parameter λ is arbiarary number. Using the standard anticommutator rules for fermionic operators $\hat{a}^\dagger_{s\mathbf{k}}$ and $\hat{a}_{s\mathbf{k}}$, and the property (8), we calculate the commutator:

$$\left[\widehat{W},\, \hat{b}^\dagger_{\mathbf{k}}(\lambda)\right] = \frac{g}{L^3} \sum_{\mathbf{k}_1 \in \mathcal{D}_F} G(\mathbf{k}_1, \mathbf{k})\, \hat{a}^\dagger_{\uparrow\mathbf{k}_1} \hat{a}^\dagger_{\downarrow-\mathbf{k}_1} \tag{11}$$

$$(1 + \lambda - \lambda\hat{a}^\dagger_{\uparrow-\mathbf{k}}\hat{a}_{\uparrow-\mathbf{k}} - \lambda\hat{a}^\dagger_{\downarrow\mathbf{k}}\hat{a}_{\downarrow\mathbf{k}} - \hat{a}^\dagger_{\uparrow\mathbf{k}}\hat{a}_{\uparrow\mathbf{k}} - \hat{a}^\dagger_{\downarrow-\mathbf{k}}\hat{a}_{\downarrow-\mathbf{k}}).$$

As is seen, when $\lambda = -1$ this commutator has the specific form, where all summands are finished by the annihilation operators $\hat{a}_{\uparrow\pm\mathbf{k}}$ and $\hat{a}_{\downarrow\pm\mathbf{k}}$. Therefore we define now the basic operator construction $\hat{\gamma}^\dagger_{\mathbf{k}}$:

$$\hat{\gamma}^\dagger_{\mathbf{k}} \equiv \hat{b}^\dagger_{\mathbf{k}}(-1) = \hat{a}^\dagger_{\uparrow\mathbf{k}} \hat{a}^\dagger_{\downarrow-\mathbf{k}} - \hat{a}^\dagger_{\uparrow-\mathbf{k}} \hat{a}^\dagger_{\downarrow\mathbf{k}}\,, \tag{12}$$

for which the commutator (11) takes the form:

$$\left[\widehat{W},\, \hat{\gamma}^\dagger_{\mathbf{k}}\right] = \frac{g}{L^3} \sum_{\mathbf{k}_1 \in \mathcal{D}_F} G(\mathbf{k}_1, \mathbf{k})\, \hat{a}^\dagger_{\uparrow\mathbf{k}_1} \hat{a}^\dagger_{\downarrow-\mathbf{k}_1}$$

$$(\hat{a}^\dagger_{\uparrow-\mathbf{k}}\hat{a}_{\uparrow-\mathbf{k}} + \hat{a}^\dagger_{\downarrow\mathbf{k}}\hat{a}_{\downarrow\mathbf{k}} - \hat{a}^\dagger_{\uparrow\mathbf{k}}\hat{a}_{\uparrow\mathbf{k}} - \hat{a}^\dagger_{\downarrow-\mathbf{k}}\hat{a}_{\downarrow-\mathbf{k}})\,. \tag{13}$$

This expression is crucial for the building up the CPT-like state (9).

It is worth to note that due to the obvious relationship

$$\hat{\gamma}^\dagger_{-\mathbf{k}} = -\hat{\gamma}^\dagger_{\mathbf{k}} \tag{14}$$

the operators $\hat{\gamma}^{\dagger}_{\mathbf{k}}$, defined on the spherical layer $\mathbf{k}{\in}\mathcal{D}_F$, are not independent. Therefore instead of the subset $\mathcal{D}_F$ we define the upper hemispherical layer $\mathcal{D}_F^{(+)}$ (see Fig.1b), consisting of vectors $\mathbf{k}{\in}\mathcal{D}_F$ with positive projections on the axis Oz ($k_z > 0$) only. Now the operators $\hat{\gamma}^{\dagger}_{\mathbf{k}}$, defined for vectors $\mathbf{k}{\in}\mathcal{D}_F^{(+)}$ will be independent.

Consider the operator construction

$$\widehat{\Psi}_{NC} = \prod_{\mathbf{k}\in\mathcal{D}_F^{(+)}} \hat{\gamma}^{\dagger}_{\mathbf{k}}\,, \tag{15}$$

which acts on the upper subset $\mathcal{D}_F^{(+)}$ (for each $\mathbf{k}$ the operator $\hat{\gamma}^{\dagger}_{\mathbf{k}}$ is used, at most, once). Obviously, the order of multipliers can be arbitrary, because $[\hat{\gamma}^{\dagger}_{\mathbf{k}}, \hat{\gamma}^{\dagger}_{\mathbf{k}'}]{=}0$. Let us factor out arbitrary operator $\hat{\gamma}^{\dagger}_{\mathbf{k}'}$ in (15) from the product Π and then act by the operator $\widehat{W}$ on $\widehat{\Psi}_{NC}$:

$$\widehat{W}\widehat{\Psi}_{NC} = \widehat{W}\hat{\gamma}^{\dagger}_{\mathbf{k}'} \prod_{\mathbf{k}\neq\mathbf{k}'} \hat{\gamma}^{\dagger}_{\mathbf{k}} = \left(\hat{\gamma}^{\dagger}_{\mathbf{k}'}\widehat{W} + [\widehat{W},\, \hat{\gamma}^{\dagger}_{\mathbf{k}'}]\right) \prod_{\mathbf{k}\neq\mathbf{k}'} \hat{\gamma}^{\dagger}_{\mathbf{k}}. \tag{16}$$

Since under the sign Π in (16) the creation operators with wavevectors $\pm\mathbf{k}'$ are absent, then, as is follows from eq.(13), the commutator $[\widehat{W},\, \hat{\gamma}^{\dagger}_{\mathbf{k}'}]$ can be moved to the right side through the product Π. As a result, the expression (16) can be written as:

$$\widehat{W}\widehat{\Psi}_{NC} = \hat{\gamma}^{\dagger}_{\mathbf{k}'}\widehat{W} \prod_{\mathbf{k}\neq\mathbf{k}'} \hat{\gamma}^{\dagger}_{\mathbf{k}} + \left(\prod_{\mathbf{k}\neq\mathbf{k}'} \hat{\gamma}^{\dagger}_{\mathbf{k}}\right) [\widehat{W},\, \hat{\gamma}^{\dagger}_{\mathbf{k}'}]\,. \tag{17}$$

Let us consider also the operator construction

$$\widehat{\Phi}(\Delta) = \prod_{|\mathbf{k}|<(k_F-\Delta)} \hat{a}^{\dagger}_{\uparrow\mathbf{k}}\hat{a}^{\dagger}_{\downarrow\mathbf{k}}\,, \tag{18}$$

which, acting on the vacuum $|0\rangle$, generates the state, corresponding to the completely occupied sphere with the radius $(k_F - \Delta)$ in the wavevector space (in Fig.1a it corresponds to the inner sphere shaded by skew lines). The following commutator relationships are evident:

$$[\widehat{W},\, \widehat{\Phi}(\Delta)] = 0; \quad \left[[\widehat{W},\, \hat{\gamma}^{\dagger}_{\mathbf{k}'}],\, \widehat{\Phi}(\Delta)\right] = 0 \quad (\mathbf{k}' \in \mathcal{D}_F)\,, \tag{19}$$

because in the operator $\widehat{\Phi}(\Delta)$ (see (18)) only the wavevectors $|\mathbf{k}|{<}(k_F{-}\Delta)$ are used. These vectors do not belong to the upper layer $\mathcal{D}_F$ where the operators $\widehat{W}$ and $\hat{\gamma}^{\dagger}_{\mathbf{k}'}$ act.

Let us prove that the state $|NC\rangle$, nullifying the interaction (9), has the form

$$|NC\rangle = \widehat{\Psi}_{NC}\widehat{\Phi}(\Delta)|0\rangle\,. \tag{20}$$

Acting on this state by the operator, and taking into account the relationships (17) and (19), one can obtain:

$$\widehat{W}\widehat{\Psi}_{NC}\widehat{\Phi}(\Delta)|0\rangle = \hat{\gamma}^{\dagger}_{\mathbf{k}'}\widehat{W}\left(\prod_{\mathbf{k}\neq\mathbf{k}'} \hat{\gamma}^{\dagger}_{\mathbf{k}}\right)\widehat{\Phi}(\Delta)|0\rangle + \left(\prod_{\mathbf{k}\neq\mathbf{k}'} \hat{\gamma}^{\dagger}_{\mathbf{k}}\right)\widehat{\Phi}(\Delta)\,[\widehat{W},\, \hat{\gamma}^{\dagger}_{\mathbf{k}'}]\,|0\rangle\,. \tag{21}$$

However, since the commutator (13) is finished from the right side by the annihilation operators, then $[\widehat{W}, \hat{\gamma}^{\dagger}_{\mathbf{k}'}]|0\rangle = 0$. Thus, from (21) we have

$$\widehat{W}\widehat{\Psi}_{NC}\widehat{\Phi}(\Delta)|0\rangle = \hat{\gamma}^{\dagger}_{\mathbf{k}'}\widehat{W}\left(\prod_{\mathbf{k}\neq\mathbf{k}'} \hat{\gamma}^{\dagger}_{\mathbf{k}}\right)\widehat{\Phi}(\Delta)|0\rangle\,, \tag{22}$$

From this equation we see that it is possible to change the sequence order of $\widehat{W}$ and any operator $\hat{\gamma}^{\dagger}_{\mathbf{k}}$. Proceeding this consideration step by step and taking into account (19), we obtain eventually:

$$\widehat{W}|NC\rangle \equiv \widehat{W}\widehat{\Psi}_{NC}\widehat{\Phi}(\Delta)|0\rangle = \widehat{\Psi}_{NC}\widehat{W}\widehat{\Phi}(\Delta)|0\rangle = \widehat{\Psi}_{NC}\widehat{\Phi}(\Delta)\widehat{W}|0\rangle = 0\,. \tag{23}$$

Here the last transformation to zero is obvious, because the operator $\widehat{W}$ (see (7)) is finished from the right side by the annihilation operators $\hat{a}_{s\pm\mathbf{k}}$. Thus, we prove rigorously that the state (20) nullifies the interparticle interaction (scattering), i.e. it obeys the equation (9).

It should be noted the presence of the construction $\widehat{\Phi}(\Delta)$ in (20) is necessary from the physical point of view, since the form of the interaction Hamiltonian (7), according to [20], is a consequence of almost completely occupied Fermi sphere. Thus, physically significant states should differ from the ideal Fermi state $|F\rangle$:

$$|F\rangle = \left(\prod_{|\mathbf{k}|\leq k_F} \hat{a}^{\dagger}_{\uparrow\mathbf{k}}\hat{a}^{\dagger}_{\downarrow\mathbf{k}}\right)|0\rangle \tag{24}$$

only in a small region nearby the Fermi sphere. For the state (20) this difference is described by the construction $\widehat{\Psi}_{NC}$ (15), acting in the thin layer $\mathcal{D}_F$ around the Fermi surface in the wavevector space.

The state $|NC\rangle$ is an eigenstate for the unperturbed Hamiltonian $\widehat{H}_0$ and, consequently, for the total Hamiltonian $\widehat{H}$:

$$\widehat{H}\,|NC\rangle = \widehat{H}_0\,|NC\rangle = E_{NC}\,|NC\rangle\,, \quad E_{NC} = \frac{3}{5}\,\mathcal{E}_F\mathcal{N}\left\{1 + 10\left(\frac{\Delta}{k_F}\right)^2 + 5\left(\frac{\Delta}{k_F}\right)^4\right\}, \tag{25}$$

where $\mathcal{E}_F{=}(\hbar k_F)^2/2m$ is the Fermi energy and $\mathcal{N}$ is the number of particles in ensemble. In the theory with chemical potential μ we should add $-\mu\mathcal{N}$ to the value E_{NC} in (25).

As to the construction $\widehat{\Psi}_{NC}$, the occupation of all the thin layer $\mathcal{D}_F$ in (15) is dictated by the conservation of particle number. Indeed, as it follows from (12), each operator $\hat{\gamma}^\dagger_{\mathbf{k}}$ describes the distribution of two electrons among the four states $|\uparrow,\mathbf{k}\rangle$, $|\downarrow,\mathbf{k}\rangle$, $|\uparrow,-\mathbf{k}\rangle$, $|\downarrow,-\mathbf{k}\rangle$. Because of this, in order to distribute all electrons, which at the density packing (into Fermi sphere) were located in the layer $(k_F-\Delta)\leq k\leq k_F$, we need in a doubled volume in the wavevector space. In the case $\Delta\ll k_F$ practically the whole thin layer $\mathcal{D}_F$ (see in Fig.1a) corresponds to a such double volume, fro which $(k_F-\Delta)\leq k\leq(k_F+\Delta)$. If the particle number conservation is not taken into account, then an arbitrary number of different operators $\hat{\gamma}^\dagger_{\mathbf{k}}$ can be used in the construction (15).

Note that the ground state in the BCS theory [20] can be written in the form

$$|BCS\rangle = \widehat{\Psi}_{BCS}\widehat{\Phi}(\Delta)|0\rangle \tag{26}$$

with the operator construction

$$\widehat{\Psi}_{BCS} = \prod_{\mathbf{k}\in\mathcal{D}_F} \left\{ u(\mathbf{k}) + v(\mathbf{k})\hat{a}^\dagger_{\uparrow\mathbf{k}}\hat{a}^\dagger_{\downarrow-\mathbf{k}} \right\}, \tag{27}$$

where $u(\mathbf{k})$ and $v(\mathbf{k})$ are variational coefficients (they are coefficients in the Bogolyubov transformation).

Let us discuss some basic properties of the state $|NC\rangle$, which are quite different from those of the state $|BCS\rangle$ in the BCS theory:

I. $|NC\rangle$ is an eigenstate for the particle number operator $\widehat{N}{=}\sum_{s,\mathbf{k}} \hat{a}^\dagger_{s\mathbf{k}}\hat{a}_{s\mathbf{k}}$:

$$\widehat{N}|NC\rangle = \mathcal{N}|NC\rangle\,. \tag{28}$$

II. The state $|NC\rangle$ is an eigenstate for the total momentum operator $\widehat{\mathbf{P}}{=}\sum_{\mathbf{k},s}(\hbar\mathbf{k})\hat{a}^\dagger_{s\mathbf{k}}\hat{a}_{s\mathbf{k}}$. For example, if the Fermi sphere is constructed around the wavevector $\mathbf{K}$, then we have:

$$\widehat{\mathbf{P}}|NC\rangle = \mathcal{N}(\hbar\mathbf{K})|NC\rangle\,, \tag{29}$$

i.e. this state corresponds to a free flow of particles. (The consideration above dealt with the particular case $\mathbf{K}{=}0$, but the generalization to arbitrary $\mathbf{K}$ is almost elementary.)

III. $|NC\rangle$ does not depend on the value and sign of the coupling constant g, i.e. it exists in both cases of weak and strong coupling, and for the case of interparticle repulsion.

IV. In the BCS theory the product $\hat{a}^\dagger_{\uparrow\mathbf{k}}\hat{a}^\dagger_{\downarrow-\mathbf{k}}$ corresponds to the operator of creation of scalar quasiparticle (Cooper pair), while in our case the construction $\hat{\gamma}^\dagger_{\mathbf{k}}$ is related to a coupled pair. This construction, as it follows from (12), generates an entangled state with respect to the spin and translational degrees of freedom.

Note, that a role of entanglement in strongly correlated spin systems (in particular for fermions in BCS) was investigated in [21, 22].

CONCLUSION

We have found the CPT-like state $|NC\rangle$ in an ensemble of fermions with spin 1/2. This state has an analogy with the CPT effect and it nullifies the interparticle interaction operator of the standard BCS theory.

Evidently, the states $|NC\rangle$ constitutes a special class of eigenstates of the total Hamiltonian $\widehat{H}$ due to the independency on the coupling constant g, while, undoubtedly, there exist other eigenstates with a nontrivial analytical g-dependence of the energy $E(g)$. Because of this a question about the physical realization of the state $|NC\rangle$ requires a separate consideration. In the case of interparticle attraction ($g<0$) the energy E_{NC} for the CPT-like state lies above the ground-state energy of the BCS theory. However, in the theory with interparticle repulsion ($g>0$) it is possible, in principle, that the state $|NC\rangle$ will be the ground state, because other states acquire a positive increment to the energy. Note that due to the Coulomb repulsion (or excitonic mechanism of interaction) between electrons the theory with interparticle repulsion has equal chances for the realization relative to the theory with attraction.

It should be stressed that, according to the property II, the state $|NC\rangle$ corresponds to a free (non-dissipative) particle flow (29) and it is a superconduting (or superfluid for ^{3}He) state just due to its nature (even without references to energy gap). From this point of view the presence of energy gap is necessary first of all for the stability of this state.

Although the problem on the realization of CPT-like states remains open, nevertheless the presented study argues for a principal possibility of alternative approaches, even in the framework of standard BCS Hamiltonian (7). It looks interesting, from our viewpoint, due to difficulties with construction of a complete theory of the high-temperature superconductivity. This, in its turn, stimulates the search of various alternatives for the BCS theory (see, for example, review [23]). The approach outlined in the present paper is another alternative based on an analogy with the CPT effect. It is advisable to study this alternative in detail in the future. Besides, it is possible that the CPT-like states will find their applications regardless to the superconductivity theory.

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